FACTORS AND MULTIPLES

A factor of a given number is that number which divides the given number without remainder. For example, all the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30 divides into 30 without a remainder.

Prime Numbers and Prime Factors

Consider 7 and 15. The factors of 7 are 1 and 7, as such 7 has only two factors, 1 and itself. A number that has only 1 and itself as factors is called a *prime number*. On the other hand, 15 has 1, 3, 5 and 15 as its factors. A number that has more that two factors is called a *composite number*. Examples of prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19, while examples of composite numbers are 4, 6, 8, 9, 10, 12, etc.

When we list the factors of any given number, we would see that some of them are prime numbers. Such factors are called prime factors. Prime factors of a nmber are those of its factors that are prime numbers.

For instance, the factors of 30 are 1, 2, 3, 5, 6, 15, and 30. Out of these, the prime numbers are 2, 3, and 5. So the prime factors of 30 are 2, 3, and 5.

To find the prime factors of a number, we divide it repeatedly with prime numbers, starting with the smallest prime number, 2. In other words, we first divide the given number by 2 until 2 cannot divide into the dividend again. We then try 3 and then 5, and then 7 and so on.

Example: express 36 as a product of its prime factors

2	**36**
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3$

Highest Common Factor

When you compare the factors of some numbers, you would see that some are common.

For example, factors of 12 are 1, 2, 3, 4, 6 and 12;
factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30; and
factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

1, 2, 3, and 6 are factors of 12, 30 and 42. We could rather say that 1, 2, 3 and 6 are common factors of 12, 30 and 42. The highest common factor is the highest of the common factors i.e. 6.

The simplest way to obtain the HCF of two or more numbers is to express each of them as a product of their prime factors, and then multiply all the common prime factors.

For example:

$12 = 2 \times 2 \times 3$
$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$

The common prime factors are 2 and 3.

$\therefore$ HCF of 12, 30 and $42 = 2 \times 3 = 6$

Lowest Common Multiples

Multiples of a particular number are the numbers that a particular number can divide without remainder.

Just as we have common factors, so we do have common multiples. For example, the multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80, 90…; the multiples of 15 are 15, 30, 45, 60, 75…; the multiples of 30 are 30, 60, 90, 120, 150….

30, 60, and 90 are common to the multiples of 10, 15, and 30. The lowest among these common multiples is 30. This method is often referred to as the *multiple method*.

The simplest way to obtain the LCM of two or more numbers is to express each of them as a product of prime factors, and then find the lowest product of the factors which contains all of the prime factors of the numbers.

For example:

10 = 2 × 5
15 = 3 × 5
30 = 2 × 3 × 5

LCM = 2 × 3 × 5 = 30

Difference between LCM and HCF

Find the LCM and HCF of 12 and 16.

LCM

2	**12**	**16**
2	6	8
2	3	4
2	3	2
3	3	1
	1	1

LCM of 12 and 16 = 2 × 2 × 2 × 2 × 3 = 48.

HCF 12 = 2 × 2 × 3
16 = 2 × 2 × 2 × 2

HCF of 12 and 16 = 2 × 2 = 4.

From the above example, you see that when HCF and LCM of the same set of numbers are calculated and compared, the LCM is always greater than the HCF.

The difference between the LCM and HCF can also be calculated, that is, $LCM - HCF = 48 - 4 = 44$